Unification of Laughlin and Moore-Read States in SUSY Quantum Hall Effect

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Based on the recently proposed SUSY quantum Hall effect, we show that Laughlin and Moore-Read states are related by a hidden SUSY transformation. Regarding the SUSY Laughlin wavefunction as a master wavefunction, Laughlin and Moore-Read states appear as two extreme limits of component wavefunctions. Realizations of topological excitations on Laughlin and Moore-Read states are also discussed in the SUSY formalism. We develop a streographically projected formulation of the SUSY quantum Hall effect. With appropriate interpretation of Grassmann odd coordinates, we illustrate striking analogies between SUSY quantum Hall effect and superfluidity.

Quantum Hall effects (QHE) have been experimentally confirmed at odd-denominator fillings and even-denominator fillings in monolayer and bilayer systems [1, 2, 3, 4]. In monolayer QHE at odd denominator filling, Laughlin wavefunction [5] well describes the groundstate. For other filling QHE, several "deformed" Laughlin wavefunctions have been proposed as groundstates. Halperin wavefunction is a two-component extension of the Laughlin wavefunction and is introduced to formulate two-spin or bilayer QH liquids [6]. Especially, Halperin 331 state is believed to describe the bilayer QH liquid at $\nu = 1/2$. Moore-Read state [7, 8] and Haldane-Rezayi state [9] are proposed as groundstate for even-denominator filling monolayer QH state. The numerical calculation [10] and the recent experiment [11] suggest that Moore-Read state is a probable groundstate at $\nu = 5/2$. Moore-Read and Haldane-Rezayi states are constructed by multiplying the pfaffian and the permanent factors to Laughlin and Halperin 331 states, respectively.

Based on the composite boson [12, 13] and composite fermion [14] picture, one may find close analogies between QHE and superfluidity. Laughlin state is regarded as "condensation" of composite bosons, analogous to ^4He superfluidity. Haldane-Rezayi state describes condensation of d-wave pairing states of composite fermions. Both Moore-Read and Halperin 331 states represent condensation of p-wave pairings of composite fermions. Besides, intriguingly, Moore-Read and Halperin 331 states are simply related by a continuous rotation of the pairing vector, and their symmetries correspond to those of the superfluid $^3He - A_1$ and $^3He - A$ phases, respectively [15]. The symmetry and superfluid analogy arguments have discovered close relations between such "different" QH liquids.

In this paper, we discuss a hidden SUSY relation between Laughlin and Moore-Read states. We use a set-up of the SUSY QHE [16, 17], which was recently proposed as a SUSY extension of the Haldane's spherical QHE [18]. The SUSY QHE contains non-anticommutative geometry as its mathematical background [19] and its low energy sector is described by a particular SUSY Chern-Simons field theory [20]. Though the SUSY QHE may not have apparent relevance to real QHE, the SUSY QHE provides an interesting unified formulation of Laughlin and Moore-Read states. Further, with appropriate interpretation of the Grassmann odd coordinates, like other QH liquids, we show the SUSY QH liquid may be understood as an exotic superfluid state.

SUSY Quantum Hall Effect— The spherical SUSY QHE is constructed on a supersphere in a supermonopole background [16]. The supersphere $S^{2|2} = OSp(1|2)/U(1)$ is a supermanifold whose coordinates satisfy the constraint $x_a^2 + C_{\alpha\beta}\theta_\alpha\theta_\beta = R^2$, where $x_a(a = 1, 2, 3)$ and $\theta_\alpha(\alpha = 1, 2)$ are Grassmann even and odd coordinates, respectively, and $C_{\alpha\beta}$ is a charge conjugation antisymmetric matrix with $C_{12} = 1$. The one-particle Hamiltonian is given by $H = \frac{1}{2MR^2}(\Lambda_a^2 + C_{\alpha\beta}\Lambda_\alpha\Lambda_\beta)$, where $\Lambda_a = -i\epsilon_{abc}x_b D_c + \frac{1}{2}\theta_\alpha(\sigma_a)_{\alpha\beta}D_\beta$ and $\Lambda_\alpha = \frac{1}{2}(C\sigma_a)_{\alpha\beta}x_a D_\beta - \frac{1}{2}\theta_\beta(\sigma_a)_{\beta\alpha}D_a$ denote the covariant angular momenta. The covariant derivatives, $D_a = \partial_a - iA_a$ and $D_\alpha = \partial_\alpha - iA_\alpha$, satisfy the algebras, $[D_a, D_b] = i\frac{I}{2R^3}\epsilon_{abc}x_c(1 + \frac{3}{2R^2}C_{\alpha\beta}\theta_\alpha\theta_\beta)$, $[D_a, D_\alpha] = \frac{I}{2R^3}(\delta_{ab} - \frac{3}{R^2}x_a x_b)(\sigma_b C)_{\alpha\beta}\theta_\beta$ and $\{D_\alpha, D_\beta\} = \frac{I}{R^3}x_a(\sigma_a C)_{\alpha\beta}(1 + \frac{3}{2R^2}C_{\gamma\delta}\theta_\gamma\theta_\delta)$. The energy eigenvalues are given by $E_n = \frac{1}{2MR^2}[n(n + \frac{1}{2}) + (n + \frac{1}{4})I]$, where $I/2$ indicates the half-integer charge of the supermonopole and n is an integer to specify Landau level. The degeneracy in n-th Landau level is given by $D_n = 4n + 2I + 1$. Especially, the degeneracy in the lowest Landau level (LLL) is $2I + 1$, and the degenerate wavefunctions are given by supermonopole harmonics which are constructed by the products of the components of the super Hopf spinor $\psi = (u, v, \eta)^t = (\sqrt{\frac{1+x_3}{2}}(1 - \frac{1}{4(1+x_3)}C_{\alpha\beta}\theta_\alpha\theta_\beta), \frac{x_1+ix_2}{\sqrt{2(1+x_3)}}(1 + \frac{1}{4(1+x_3)}C_{\alpha\beta}\theta_\alpha\theta_\beta), \sqrt{\frac{1+x_3}{2}}\theta_1 + \frac{x_1+ix_2}{\sqrt{2(1+x_3)}}\theta_2)^t$ as

$$u^{I-p}v^p, \quad u^{I-q-1}v^q\eta,$$

with $0 \leq p \leq I$, $0 \leq q \leq I - 1$. Their eigenvalues of L_3 are given by $\frac{I}{2} - p$ and $\frac{I}{2} - q - \frac{1}{2}$, respectively. The spherical SUSY Laughlin wavefunction is derived as

$$\Psi = \prod_{i<j}^{N}(u_i v_j - v_i u_j - \eta_i \eta_j)^m,$$

and it is invariant under the $OSp(1|2)$ transformation generated by $L_a = \psi^t l_a \frac{\partial}{\partial \psi}$ and $L_\alpha = \psi^t l_\alpha \frac{\partial}{\partial \psi}$. Here, l_a and l_α are given by $l_a = \frac{1}{2}\begin{pmatrix} \sigma_a & 0 \\ 0 & 0 \end{pmatrix}$, $l_\alpha = \frac{1}{2}\begin{pmatrix} 0 & \tau_\alpha \\ -(C\tau_\alpha)^t & 0 \end{pmatrix}$, where $\tau_1 = (1,0)^t$ and $\tau_2 = (0,1)^t$. Since the original Laughlin wavefunction takes the form [18]

$$\Phi = \prod_{i<j}(u_i v_j - v_i u_j)^m,$$

the only difference between Ψ and Φ is the $\eta\eta$ term [21]. As the SUSY system generally contains both bosonic and fermionic states, the SUSY QHE at $\nu = 1/m$ is doubly degenerate compared to the original "bosonic" QHE at $\nu = 1/m$ [16, 17].

Here, we introduce interesting correspondences between the SUSY QHE and a projected original QHE which we call the "vector" QHE. We focus on the original Haldane's system subject to *even* Landau levels in the background of Dirac monopole with *integer* charge. In such projected Haldane's system, the energy eigenvalues are given by $\epsilon_{2n} = 4 \times \frac{1}{2MR^2}(n(n+\frac{1}{2}) + I(n+\frac{1}{4}))$, which are exactly equivalent to the energy spectra for the above SUSY Landau problem up to the proportional factor 4. Besides, the degeneracy in $2n$-th Landau level is given by $d_{2n} = 4n + 2I + 1$, which is again equal to the degeneracy of n-th SUSY Landau level. Thus, the original Landau problem subject to even Landau levels with integer monopole charge has *quantitative* correspondence to the SUSY Landau problem. The degeneracy of the present LLL is $d_0 = 2I+1$, and the degenerate eigenstates are given by the vector monopole harmonics

$$U^{I-p}V^p, \quad U^{I-q-1}V^q W,$$

where $0 \le p \le I$, $0 \le q \le I-1$, and $(U,W,V) = (u^2, \sqrt{2}uv, v^2)$ denotes the SO(3) Hopf vector. The eigenvalues of L_3 for the above vector monopole harmonics are, respectively, given by $I - 2p$ and $I - 2q - 1$. One may find apparent analogies between the vector monopole harmonics and the super monopole harmonics on the correspondence between the Hopf vector (U, V, W) and the SUSY Hopf spinor (u, v, η). Since $SO(3)$ singlet is made as the symmetric combination of two vectors, the "vector" Laughlin wavefunction is constructed as

$$\Phi_v = \prod_{i<j}(U_i V_j + V_i U_j - W_i W_j)^m = \prod_{i<j}(u_i v_j - u_i v_j)^{2m}.$$

Again, there is a formal resemblance between Ψ and Φ_v by the correspondence between (U, V, W) and (u, v, η). The vector Laughlin wavefunction is simply equal to the original Laughlin wavefunction at the even denominator filling $\nu = 1/2m$, which contains double states compared to the original system at $\nu = 1/m$. This may reminds the doubly degenerate feature of SUSY system. Thus, at the

many-body wavefunction level, the vector QHE possesses several qualitative analogies to the SUSY QHE.

Expansion of SUSY Laughlin State.— We expand the SUSY Laughlin wavefunction in terms of the Grassmann odd quantities as in the superfield formalism. With the composite quantity

$$Q = \sum_{i<j} \frac{\eta_i \eta_j}{u_i v_j - v_i u_j},$$

which we call the pairing operator, the SUSY Laughlin wavefunction is simply rewritten as

$$\Psi = e^{-mQ} \cdot \Phi = \sum_{k=0}^{N/2} (-m)^k \Psi_k.$$

In the last equation, we expanded the exponential in terms of Q, and the component wavefunctions Ψ_k ($k = 1, 2, \cdots, N/2$) are given by

$$\Psi_k = \frac{1}{k!} Q^k \cdot \Phi.$$

Explicitly, they are

$$\Psi_0 = \Phi,$$
$$\Psi_1 = \sum_{i<j} \eta_i \eta_j \frac{1}{\phi_{ij}} \cdot \Phi,$$
$$\Psi_2 = \sum_{i<j<k<l} \eta_i \eta_j \eta_k \eta_l \frac{1}{\phi_{ijkl}} \cdot \Phi,$$
$$\vdots$$
$$\Psi_{N/2} = \eta_1 \eta_2 \cdots \eta_N \cdot Pf(\frac{1}{\phi_{ij}}) \, \Phi,$$

where $\frac{1}{\phi_{ij}} \equiv \frac{1}{u_i v_j - v_i u_j}$, $\frac{1}{\phi_{ijkl}} \equiv \frac{1}{\phi_{ij}\phi_{kl}} - \frac{1}{\phi_{ik}\phi_{jl}} + \frac{1}{\phi_{il}\phi_{jk}}$. In the expression of $\Psi_{N/2}$, we used the formula $\eta_{i_1} \eta_{i_2} \cdots \eta_{i_N} = \epsilon_{i_1 i_2 \cdots i_N} \eta_1 \eta_2 \cdots \eta_N$. It is noted that the original Laughlin wavefunction appears as the 1st component wavefunction Ψ_0, and, remarkably, Moore-Read state appears as the last component wavefunction $\Psi_{N/2}$. This expansion suggests that Laughlin and Moore-Read states are related by a SUSY transformation. To clarify the direct SUSY relation between Laughlin and Moore-Read states, it is important to explore the properties of the pairing operator. While Q is a $SU(2)$-singlet quantity, it is not invariant under the Grassmann odd transformation generated by L_α. However, as is easily checked, Q returns to itself under the two successive operations of L_α,

$$C_{\alpha\beta} L_\alpha L_\beta Q = \frac{1}{2} Q - \frac{1}{4} N(N-1).$$

Besides, since the SUSY Laughlin wavefunction $\Psi = e^{-mQ}\Phi$ is invariant under the $OSp(1|2)$ transformation, Φ and Q satisfy the relation $C_{\alpha\beta} L_\alpha L_\beta \ln \Phi =$

$mC_{\alpha\beta}L_\alpha L_\beta Q$. These relations suggest that Q is generated by the two successive SUSY transformations from Φ,

$$Q = \frac{2}{m}C_{\alpha\beta}L_\alpha L_\beta \ln\Phi + \frac{N(N-1)}{2}.$$

With this relation, every component wavefunction can be represented by the SUSY transformation of the original Laughlin wavefunction. For instance, the 2nd component wavefunction is expressed as $\Psi_1 = Q\Phi = \frac{2}{m}C_{\alpha\beta}L_\alpha L_\beta\Phi - \frac{2}{m}C_{\alpha\beta}(L_\alpha\ln\Phi)(L_\beta\ln\Phi)\Phi + \frac{N(N-1)}{2}\Phi$. Repeating such SUSY transformation $N/2$ times, Moore-Read state $\Psi_{N/2}$ is finally constructed from the Laughlin wavefunction Φ as

$$\Psi_{N/2} = \frac{1}{(N/2)!}Q^{\frac{N}{2}}\Phi$$
$$= \frac{1}{(N/2)!}\left(\frac{2}{m}\right)^{\frac{N}{2}}(C_{\alpha\beta}L_\alpha L_\beta)^{\frac{N}{2}}\Phi + (\text{non-linear terms}).$$

Thus, Laughlin and Moore-Read states are related by the nonlinear SUSY transformation generated by L_α.

Topological Excitations— The quasi-hole excitation at the point $(\Omega_a, \Omega_\alpha) = (2\chi^\ddagger l_a\chi, 2\chi^\ddagger l_\alpha\chi)$ with $\chi = (a, b, \xi)^t$ and $\chi^\ddagger = (a^*, b^*, -\xi^*)$ on the supersphere is generated by the operator $\mathcal{A}^\ddagger = \prod_i \mathcal{A}_i^\ddagger$, where $\mathcal{A}_i^\ddagger = av_i - bu_i - \xi\eta_i$ [16]. It is apparent, without $\xi\eta$ term, $A_i^\ddagger$ is reduced to $A_i^\dagger = av_i - bu_i$, which is the quasi-hole operator on the original Laughlin state. The fundamental excitation on the Moore-Read state is called the halberon, which carries half of the electric charge of a naively expected quasi-particle excitation [8]. The halberon pair wavefunction is given by $Pf\left(\frac{A_i^\dagger A_j'^\dagger + A_j^\dagger A_i'^\dagger}{\phi_{ij}}\right)\cdot\Phi$. In the SUSY formalism, the halberon pair operator is constructed as $\mathcal{A}_{h.b.}^\ddagger = \prod_{i<j}\exp\left(m\frac{\eta_i\eta_j}{\psi_{ij}}(1 - \mathcal{A}_i^\ddagger\mathcal{A}_j'^\ddagger - \mathcal{A}_j^\ddagger\mathcal{A}_i'^\ddagger)\right)$, where $\psi_{ij} = u_iv_j - v_iu_j - \eta_i\eta_j$. Indeed, this operator acts the SUSY Laughlin wavefunction as

$$\mathcal{A}_{h.b.}^\ddagger\Psi = \prod_{i<j}\exp\left(-m\frac{\eta_i\eta_j}{\phi_{ij}}(A_i^\dagger A_j'^\dagger + A_j^\dagger A_i'^\dagger)\right)\cdot\Phi,$$

and $\mathcal{A}_{h.b.}^\ddagger\Psi$ yields the halberon pair wavefunction as the last term of its expansion.

The Stereographic Projection— By the stereographic projection from the supersphere to the superplane, we construct a planar SUSY QHE. The stereographic super-coordinates (z, θ) are defined as $z \equiv \frac{v}{u} = \frac{x_1 + ix_2}{R+x_3}(R + \frac{1}{2(R+x_3)}C_{\alpha\beta}\theta_\alpha\theta_\beta)$ and $\theta \equiv \frac{\eta}{u} = \theta_1 + \frac{1}{R}z\theta_2$. z and θ respectively indicate bosonic and fermionic complex coordinates on the superplane $R^{2|2}$. With use of the stereographic super-coordinates, the super Hopf spinor is rewritten as $\psi = (u, v, \eta)^t = \frac{1}{\sqrt{R^2+zz^*}}(R, z, \theta)^t(1 - \frac{1}{2(R^2+zz^*)}\theta\theta^*)$, and in the thermodynamic limit $R, I \to \infty$

with $B = \frac{I}{2R^2}$ fixed, the supermonopole harmonics are reduced to

$$u^{I-p}v^p \to z^p\exp(-B(zz^* + \theta\theta^*)),$$
$$u^{I-q-1}v^q\eta \to z^q\theta\exp(-B(zz^* + \theta\theta^*)).$$

These expressions are consistent with the previously derived LLL basis of the planar SUSY Landau problem [17]. Similarly, the planar SUSY Laughlin wavefunction is derived as

$$\Psi = \prod_{i<j}(z_i - z_j + \theta_i\theta_j)^m \cdot \exp\left(-\frac{1}{\ell_B^2}\sum_i(z_iz_i^* + \theta_i\theta_i^*)\right),$$

where $\ell_B = R\sqrt{\frac{2}{mI}}$. Some comments are added here. While the spherical SUSY Laughlin wavefunction is expressed as the bilinear form about Grassmann even and odd quantities, the planar SUSY Laughlin wavefunction is linear for the bosonic coordinate z and bilinear for the fermionic coordinate θ (up to the exponential term). As we shall see below, this discrepancy generates magnetic translation asymmetry about the bosonic and the fermionic directions in the planar case. The $OSp(1|2)$ generators L_a and L_α correspond to the magnetic translation generators $(X, Y, \Theta_1, \Theta_2)$ and the perpendicular angular momentum $L_\perp$ in the super-plane case. Physically, $(X, Y, \Theta_1, \Theta_2)$ represent the center of mass coordinates on the superplane, and are defined as $(X, Y, \Theta_1, \Theta_2) = (x - i\ell_B^2 D_y, y + i\ell_B^2 D_x, \theta_1 + \ell_B^2 D_{\theta_2}, \theta_2 + \ell_B^2 D_{\theta_1})$, where the covariant derivatives $D_x = \partial_x - iA_x, D_y = \partial_y - iA_y, D_{\theta_1} = \partial_{\theta_1} - iA_{\theta_1}$ and $D_{\theta_2} = \partial_{\theta_2} - iA_{\theta_2}$ satisfy the relation $[D_x, D_y] = i\{D_{\theta_1}, D_{\theta_2}\} = -iB$. $L_\perp$ is given by $L_\perp = L_\perp^B + L_\perp^F$, where $L_\perp^B = -i\epsilon_{ij}x_i\frac{\partial}{\partial x_j}$ and $L_\perp^F = \frac{1}{2}\theta_\alpha(\sigma_3)_{\alpha\beta}\frac{\partial}{\partial\theta_\beta}$. $L_\perp^B$ and $L_\perp^F$ represent the *orbital* and *spin* angular momentum operators, respectively [17]. In the symmetric gauge, these generators are represented as $X = \frac{1}{2}(z + z^* + \ell_B^2\frac{\partial}{\partial z} - \ell_B^2\frac{\partial}{\partial z^*}), Y = i\frac{1}{2}(-z + z^* + \ell_B^2\frac{\partial}{\partial z} + \ell_B^2\frac{\partial}{\partial z^*}), \Theta_1 = \frac{1}{\sqrt{2}}(\theta + \ell_B^2\frac{\partial}{\partial\theta^*}), \Theta_2 = \frac{1}{\sqrt{2}}(\theta^* + \ell_B^2\frac{\partial}{\partial\theta})$ and $L_\perp = z\frac{\partial}{\partial z} - z^*\frac{\partial}{\partial z^*} + \frac{1}{2}(\theta\frac{\partial}{\partial\theta} - \theta^*\frac{\partial}{\partial\theta^*})$. With these expressions, it is straightforward to prove that the planar SUSY Laughlin wavefunction is invariant only for the translations generated by $X - iY$, $(X - iY)\Psi = 0$, and $L_\perp$, $L_\perp\Psi = m\frac{N(N-1)}{2}\Psi$. The stereographic projection breaks the translation symmetries generated by $X + iY$ and all fermionic generators Θ_1 and Θ_2 [22]. The planar SUSY Laughlin wavefunction is zero energy state of the hard-core Hamiltonian

$$H_{HC} = \sum_{l<m}V_l P_l,$$

where l is the integer which indicates the eigenvalue of the dimensionless super-radius operator $\frac{1}{2\ell_B^2}(X_i^2 + C_{\alpha\beta}\Theta_\alpha\Theta_\beta)$ between arbitrary two-particles.

Replacing the pairing operator with its planar form

$$Q = -\sum_{i<j}\frac{\theta_i\theta_j}{z_i - z_j},$$

the planar SUSY Laughlin wavefunction has same expansion as of the spherical SUSY Laughlin wavefunction. The pairing operator and the angular momentum operators satisfy the relations,

$$[L_\perp^B, Q] = -Q, \quad [L_\perp^F, Q] = Q, \quad [L_\perp, Q] = 0.$$

The physical meaning of Q is clear in this planar form. θ is the eigenstate of the spin operator $L_\perp^F$ with the eigenvalue $1/2$, and may be interpreted as the $1/2$ spin-up state. (Similarly, θ^* may corresponds to the $1/2$ spin-down state.) Meanwhile, $1/z$ represents a p-wave boundstate, since $1/z$ carries the orbital angular momentum -1, and is the eigenstate of the two-dimensional Schröedinger equation of the delta-function type attractive potential. Then, when Q operates the Laughlin state, the numerator $\theta_i \theta_j$ attaches $1/2$ up-spins to two spin-less fermions i and j, and the denominator $\frac{1}{z_i - z_j}$ acts to form a p-wave pairing state of such two particles. Since Ψ_1 is constructed by multiplying Q to Φ once, Ψ_1 contains one p-wave pairing state with polarized spins on Laughlin state. Similarly, in Ψ_2, Q is multiplied to Φ twice, and two p-wave pairing states with polarized spins are formed on the Laughlin state. Repeating this procedure, we finally obtain the last component wavefuncton $\Psi_{N/2}$, in which all particles form p-wave pairing states with polarized spins [Fig.1]. This p-wave pairing superfluid state is indeed the Moore-Read state, and $\Psi_{N/2}$ even catches the polarized spin structure of Moore-Read state. The k-th component wavefunction Ψ_k is an eigenstate of both orbital angular momentum $L_\perp^B$ and spin angular momentum $L_\perp^F$ as

$$L_\perp^B \Psi_k = (m\frac{N(N-1)}{2} - k)\Psi_k,$$
$$L_\perp^F \Psi_k = k\Psi_k.$$

These relations support the above physical interpretation of the component wavefunctions. Since the original Laughlin state and the k p-wave pairing states have the orbital angular momenta, $m\frac{N(N-1)}{2}$ and $k \times (-1)$, respectively, the total orbital angular momentum of Ψ_k is given by $m\frac{N(N-1)}{2} - k$. Similarly, since there are k pairs of polarized spins on the Laughlin state, the total spin angular momentum of Ψ_k is $\frac{1}{2} \times 2k$. Though the component wavefunctions carry different orbital and spin angular momenta, each of them carries the identical angular momentum, $m\frac{N(N-1)}{2}$, in total.

Analogy to Superfluidity.— The above expansion of the SUSY Laughlin state suggests close analogies between the superfluidity and the SUSY QHE. The BCS wavefunction has the form, $|BCS> = \prod_k (1 + g_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger)|0>$, where g_k is the coherence factor and $c_{k\sigma}^\dagger$ indicates the creation operator for electron with momentum k and spin σ, $c_{k\sigma}^{\dagger\ 2} = 0$, $\{c_{k\sigma}, c_{k'\sigma'}^\dagger\} = \delta_{kk'}\delta_{\sigma\sigma'}$. BCS state can be expressed as the superposition of the number states of Cooper pairs [23], $|BCS> = e^{\sum_k g_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger}|0> = |0>$
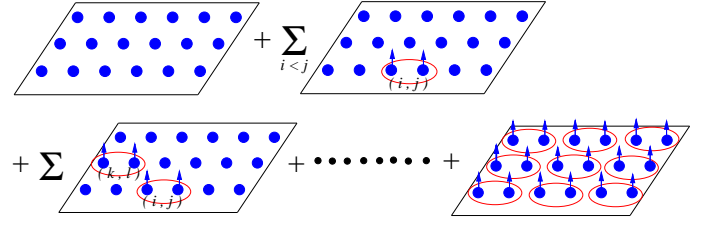

FIG. 1: The graphical representation of the expansion of the SUSY Laughlin wavefunction.

$+ \sum_k g_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger |0> + \cdots + \prod_k g_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger |0>$. Comparing the expansion formulas of BCS state and SUSY Laughlin state, one may find striking analogies between quantities in such two states; the vacuum $|0> \leftrightarrow \Phi$, the pairing operator $c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \leftrightarrow \frac{\theta_i \theta_j}{z_i - z_j}$, and the coherence factor $g_k \leftrightarrow 1$. (The factor m on the SUSY Laughlin wavefunction is simply a scaling factor, and should not be taken as the coherence factor.) While the BCS vacuum is the electron zero number state, the corresponding "vacuum" of the SUSY QHE is not the 0-particle state but the original Laughlin state. The pairing operator in SUSY Laughlin wavefunction is interpreted as the p-wave pairing operator with adding spin-up degrees of freedom. Thus, the SUSY Laughlin wavefunction is regarded as an exotic p-wave pairing superfluid state on the vacuum Laughlin state.

The SUSY Laughlin wavefunction carries two important aspects, one of which is QH aspect and the other is BCS aspect. When we define the fermion number operator $F = \sum_i (\theta_i \frac{\partial}{\partial \theta_i} - \theta_i^* \frac{\partial}{\partial \theta_i^*})$, the original Laughlin state is the vacuum of the fermion $F\Phi = 0$, and is the minimum fermion number state. Meanwhile, our Moore-Read state $\Psi_{N/2}$ has the maximum fermion number as $F\Psi_{N/2} = N\Psi_{N/2}$. Then, the bosonic limit corresponds to the original Laughlin state, which reflects the original QH aspect, while the fermionic limit corresponds to the Moore-Read state, which reflects BCS fermion pairing aspect. The coherence factor g_k in the BCS wavefunction represents the ratio of the amplitudes between the 0-particle state and the occupied state. When the coherence factor takes unity, the BCS state exists at the "middle" between the 0-particle and the occupied states, and the particle number fluctuation is maximized. As seen above, the coherence factor in the SUSY QHE reads as unity, which means the SUSY Laughlin wavefunction has same contributions from bosonic and fermionic sides. This indicates that the SUSY Laughlin wavefunction possesses the supersymmetry. Thus, the existence of the SUSY in the present QH system could be physically "translated" in the language of the superfluidity.

Though the Laughlin and the Moore-Read states belong to different topological order classes, they are related as component wavefunctions in a single SUSY Laughlin wavefunction. The SUSY extension seems to circumvent the conventional no-go topological order arguments. The connections between the supersymmetrization and the

topological order should be pursued in a future research.

Finally, we point out interesting analogies between the SUSY QHE and the supertwistor theory. In both twistor and spherical QHE contain the Hopf map as a crucial ingredience of thier constructions [24]. Further, the Grassmann coordinates in the supertwistor theory are also interpreted as spin degrees of freedom, and the component wavefunctions of the SUSY Laughlin wavefunction correspond to the twistor functions in the supertwistor formalism [25]. It would be worthwhile to speculate implications of the correpondences between these two theories.

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